

# Nonsingular vortices in (s+d)-wave superconductors

A. S. Mel'nikov, I. M. Nefedov, D. A. Ryzhov, I. A. Shereshevskii, P. P. Vysheslavtsev \* <sup>a</sup>

<sup>a</sup>Institute for Physics of Microstructures, Russian Academy of Sciences  
603600, Nizhny Novgorod, GSP-105, Russia

The structure of a single flux line in (s+d)-wave superconductors has been analyzed within the Ginzburg-Landau (GL) model generalized for two order parameter components. The fourfold symmetric singular vortex solution is shown to be unstable in a certain range of the GL parameters with respect to the mutual shift of s- and d- wave unit vortices. The resulting nonsingular vortex structure is studied both analytically and numerically.

Recently the distinctive characteristics of vortices in unconventional superconductors which can be described by the phenomenological Ginzburg-Landau (GL) theory with a multicomponent order parameter (OP) are of great interest in connection with the investigations of the mixed state structure in high- $T_c$  and heavy fermion compounds which are strong candidates for unconventional superconductivity. A single flux line in such systems is known to contain a set of unit vortices of different OP components [1–4]. One can identify two possible types of flux lines: (i) singular vortices (which have at least one point where the superconducting gap is zero) and (ii) nonsingular vortices (where the gap is nonzero everywhere in the vortex core). In this paper we focus on the case of  $s + d_{x^2-y^2}$ -wave pairing (which can be relevant to the case of high- $T_c$  superconductors) and consider the range of GL functional parameters where the singular vortices (studied in [2–4]) become unstable according to the scenario analogous to the one proposed in [1,2] for heavy fermion compounds. The goal of this paper is to analyse the detailed structure of resulting nonsingular vortices using both numerical and analytical methods.

We start with the GL free energy functional generalized for two components of the OP  $\Psi_d$  and  $\Psi_s$  corresponding to the  $d_{x^2-y^2}$ -wave and s-wave

pairing, respectively [4]:

$$F = \int \left\{ a_d |\Psi_d|^2 + a_s |\Psi_s|^2 + \frac{b_d}{2} |\Psi_d|^4 + \frac{b_s}{2} |\Psi_s|^4 + \alpha |\Psi_d|^2 |\Psi_s|^2 + \frac{\beta}{2} (\Psi_d^2 \Psi_s^{*2} + \Psi_d^{*2} \Psi_s^2) + K_s |\mathbf{\Pi} \Psi_s|^2 + K_d |\mathbf{\Pi} \Psi_d|^2 + \gamma [(\mathbf{\Pi}_x^* \Psi_s^* \mathbf{\Pi}_x \Psi_d - \mathbf{\Pi}_y^* \Psi_s^* \mathbf{\Pi}_y \Psi_d) + c.c.] + \frac{\mathbf{H}^2}{8\pi} \right\} d\mathbf{r}, \quad (1)$$

where  $\mathbf{\Pi} = \nabla - i \frac{2\pi}{\Phi_0} \mathbf{A}$ ,  $\mathbf{H} = \text{curl} \mathbf{A}$ ,  $\mathbf{r} = (x, y)$ ,  $a_s = \alpha_s (T - T_{cs})$ ,  $a_d = \alpha_d (T - T_{cd})$  (we assume  $T_{cs} < T_{cd}$  and the magnetic field is applied parallel to the  $c$ -axis). If the GL parameters are chosen so that there is an one-component homogeneous state ( $\mathbf{H} = 0$ ), then for  $\mathbf{H} \neq 0$  the subdominant OP can appear in the vortex core regions with the inhomogeneous dominant OP component either due to the mixing gradient terms ( $\gamma \neq 0$ ) [3,4] or due to the instability of the inhomogeneous state against the formation of the subdominant OP nucleus [1,2,5]. Let us consider the latter mechanism (which is obviously responsible for the formation of nonsingular vortices) for the simplest case  $\gamma = 0$ ,  $\beta = 0$  and continue with the analysis of the linearised GL equation of  $\Psi_s$ :

$$-K_s \nabla^2 \Psi_s + \alpha |\Psi_d|^2 \Psi_s = -a_s \Psi_s, \quad (2)$$

where  $\Psi_d$  describes the vortex solution within the conventional single-component GL theory, also we neglect  $\mathbf{A}$ . The "lowest energy eigenstate" of the Schrödinger equation (2) defines

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the temperature  $T^*$  of the phase transition into nonsingular vortex state. Using the following approximation  $|\Psi_d| = |\Psi_d|_\infty r / \sqrt{r^2 + 2\xi_d^2}$  (here  $|\Psi_d|_\infty = \sqrt{|a_d|/b_d}$ ,  $\xi_d = \sqrt{K_d/|a_d|}$ ) for the cases  $K_s \ll K_d a_s/a_d$  and  $\alpha \ll b_d K_s/K_d$ , we obtain  $-a(T^*) = \sqrt{2K_s \alpha / (K_d b_d)}$  and  $-a(T^*) \simeq \alpha/b_d$ , respectively (here  $a(T) = a_s(T)/|a_d(T)|$ ). We propose the following extrapolation for the phase transition curve on  $\alpha - T$  plane (Fig.1), which separates the regions with singular and nonsingular vortices:  $\alpha = -b_d a(T^*)(1 - a(T^*)K_d/(2K_s))$ . This expression is valid for the above two limiting

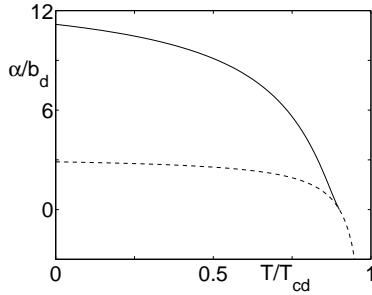


Figure 1. The phase diagram on  $\alpha - T$ -plane for  $\alpha_s = 3.2\alpha_d$ ,  $T_{cs} = 0.9T_{cd}$ ,  $b_s = 9b_d$ ,  $\beta = 0$ ,  $K_s = 0.5K_d$ . The homogeneous states with  $\Psi_s = 0$ ,  $\Psi_d \neq 0$  and with  $\Psi_s, \Psi_d \neq 0$  exist above and below the dashed line, respectively. The nonsingular vortices exist between the dashed line and the solid line.

cases and for the particular case ( $\alpha_s = 1.25\alpha_d$ ,  $T = 0$ ,  $T_{cs} = 0.8T_{cd}$ ,  $b_s = b_d$ ,  $\beta = 0$ ,  $K_s = K_d$ ,  $\gamma = 0$ ) studied in [1]. The analysis of the structure of nonsingular vortices has been carried out using numerical calculations based on the time-dependent GL theory. It was found out that for  $\gamma \neq 0$  the nonsingular vortices with broken fourfold symmetry (see Fig.2) are stable below the phase transition curve (solid line in Fig.1), whereas above the curve it is the singular fourfold symmetric vortices that are energetically favourable.

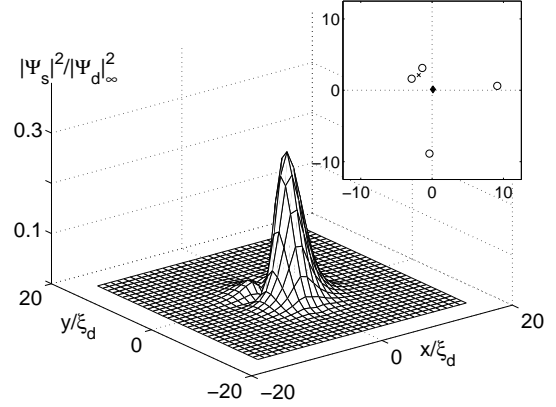


Figure 2. The typical structure of s-wave OP for nonsingular vortex for  $\alpha_s = 3.2\alpha_d$ ,  $T_{cs} = 0.95T_{cd}$ ,  $T = 0.68T_{cd}$ ,  $b_s = 9b_d$ ,  $\alpha = 3b_d$ ,  $\beta = 0$ ,  $K_s = 0.5K_d$ ,  $\gamma = 0.65K_d$ . The inset shows the positions of d-wave vortex whose winding number  $N = +1$  (black diamond), the s-wave vortices with  $N = +1$  (circles) and with  $N = -1$  (cross).

In conclusion, we have considered the structure of nonsingular vortices in  $s + d_{x^2-y^2}$  superconductor and found the range of parameters where these vortices exist. The nonsingular vortices in such materials have neither the fourfold symmetry, nor the normal core region. This can lead to some interesting effects (e.g. nontrivial dynamics, moderate pinning effects etc.).

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